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ORIGINAL ARTICLE

Determination of the Optimal State Variables in the Weighted Least Square Estimator

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ABSTRACT

This paper proposes a new criterion for choosing the optimal state variables from the stages of the weighted least squares algorithm in power systems state estimation. Accordingly, it is shown that by reduction changes of the state variables, sum of the weighted square errors will not be decreased. In other words, it is possible that the minimum amount of the sum of weighted square errors is created in the earlier stage before the final stage. This means that the estimation amounts of the system state variables might be closer to the measured values in the stage before the final stage, and instead of the choosing the state variables corresponding to the final stage, the stage in which the sum of weighted square errors has the minimum amount should be chosen. Proposed method is tested on a hypothetical 7 bus system and IEEE 14 bus test system.
Keywords: *State Estimate, Weighted Least Square (WLS), State Variable, Covariance Matrix.*

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INTRODUCTION

In recent years, the role of power system state estimator in establishing a real-time control system in energy control centers has been recognized by more and more utilities. State estimator is very necessary in establishing a complete, reliable data base for power system real-time computer applications [1].

So far different algorithms to estimate the state of power systems have been presented. The algorithm is divided to the two major intelligent methods and mathematic based techniques. Intelligent models have a reasonable speed than the mathematical models [2. 3, 4, 5, 6], but their accuracy is less than mathematical models due to the difficulty in training intelligent models in the different network situations. Mathematical models presented so far includes; the weighted least square errors, the weighted least absolute value, non-quadratic estimators and the least median squares. Among the mathematical models, the first two models have good accuracy [7, 8]. Of course, the first model has better speed than the second model and will be used more [9, 10].

The idea of using WLS was presented first by Fred Schweppe [11]. This method is based on minimizing weighted sum of squares of the difference between measured values and the calculated values. Since in the AC systems, the relations between the injected power at buses and transmission lines power with state variables (voltages and relative angles of buses) is non-linear, the estimated values are calculated based on iterative methods such as Newton-Raphson method. This iterative process will be continued until the changes amount of state variables will be smaller than a small amount such as ε. In this case, the value of sum of weighted squares difference between calculated values and the measured values will be obtained and evaluated as calculation accuracy.

This paper shows that although state variables change value has been reduced during calculation process, not necessarily sum of weighted squares errors has the minimum amount in the final stage, and it is possible that the least amount of error is created in stages other than the final stage. In other words, it is possible that the state variables in non final stage are estimated as the best state variables.

PROBLEM FORMULATION

A. State Estimation

The objective of state estimation in electric power system is to find the estimate $\hat{x}$ of the true state x which best fits the measurements z related to x through the nonlinear model [12]:

$$z = h(x) + e \tag{1}$$

Where

z is the m-dimensional measurement vector,

x is the n-dimensional state vector of voltage magnitudes and phase angles,

$h(x)$ is the vector with non-linear functions,

e is the m-dimensional error vector

The state estimation procedure by weighted least square (WLS) method involves finding the n dimensional state vector x those results from minimizing the following equation [13]:

$$J(x) = [z - h(x)]^T W [z - h(x)] \tag{2}$$

Where $W=Rz^{-1}$ is a diagonal matrix whose elements are the inverse of the covariance matrix of measurements (Rz). The state estimate ($\hat{x}$) is obtained by the following iterative procedure [14]:

$$x^{k+1} = x^k + G^{-1}(x^k)\, H^T(x^k) W [z - h(x^k)] \tag{3}$$

Where $G(x^k) = H^T(x^k)WH(x^k)$ is the gain matrix and $H=dh(x)/dx$ is.

So, given that in the AC systems state estimation solution is iterative, optimization algorithms is the Fig. 1.

It might be thought that by increasing iterations steps and reducing $max|\Delta x|$, the value of $J(x)$ will be decreased but the fact is that it may be not the case like Fig. 2. Therefore, the operator get a confusion that which parts of the points 1 or 2 is a final answer to the problem?

Although it is clear that our objective is minimizing $J(x)$ but we cannot select a stop criterion based on it. In other words, it cannot be recognized no longer that in which stages the value of $J(x)$ would be minimized and based on it algorithm will be stopped. Also, selecting the final stage $J(x)$ violates the aim which is minimizing the amount of $J(x)$. The proposed idea in this paper is that the iteration stages of the algorithm will be continued until the $max|\Delta x|>\varepsilon$ is satisfied, and then algorithm cannot find more accurate solution for the state variables than ε. But at each stage the amount of $J(x)$ and x will be saved after terminating the iterative stages and the $max|\Delta x|<\varepsilon$ would be satisfied, the $J(x)$ minimum value and the corresponded x selected as the problem solution.

Another question is that why the value of $J(x)$ might be increased by reducing $max|\Delta x|$?

The answer can be analyzed by a simple instance such as Fig. 3.

In Fig. 3, it is assumed that:

The function f is dependant to a variable x. (such as $J(x)$).

Z_m is a value observed by a microscope or any other hypothetical instrument. (such as the measured amount in power systems).

Optimization algorithm started with an assumed value such as x corresponding to the point Z_0. Clearly, it is possible that after n iteration steps the estimated variable value will be equal to x_1 and after m iterations will be equal to x_2 in which $m> n$ and $|x_m-x_2|> \varepsilon$.

Thus, according to Fig. 3 $|x_m-x_1|>|x_m-x_2|$ But $f(x_1)<f(x_2)$ and this confirms the problem assumption. However, the actual amount may be a bit different from the Z_m and the above condition would not be established, but considering the fact that the actual amount is not available for operator, therefore minimizing objective is the difference between the estimated and the measured amounts.

CASE STUDIES

For case studies the 7-buses hypothetical network reference [15] and 14-buses IEEE network reference [16] has been tested. Measured, actual and estimated data have been presented in Appendix A. Results have been achieved according to Table I and Figs. (4) and (5) too.

It can be seen that for the IEEE 14 bus test system $J(x)$ value in step 23, and for 7-buses network reference [15], at least 20 steps are repeated while the number of algorithm iterations for abovementioned networks are 79 and 59, respectively.

TABLE 1. STATE ESTIMATION RESULTS IN WLS METHOD IN 7-BUSES SYSTEM REFERENCE [15] AND IEEE 14 BUS TEST SYSTEM

Objective function value		**14 –IEEE buses**	**7-buses reference [17]**
J(x)min	Value	10.2786	40.5075
	Iterative stages	6	20
J(x)final	Value	11.7290	43.9310
	The number of iterations	111	326

TABLE 2. RESULTS OF ES FOR 7 BUSES SYSTEM

Measurement	**Measured data**	**Final Estimated data**	**Actual data**	**Optimal Estimated data**	**Measurement**	**Measured data**	**Final Estimated data**	**Actual data**	**Optimal Estimated data**
Unit	*	*	*	*	Unit	*	*	*	*
V1	238.4	243.517	241.5	240.6646	Q2,1	9.7	11.6152	12.8	11.6759
V2	237.8	242.6127	241.5	239.7324	Q2,4	38.3	45.4604	46.1	45.6579
V3	250.7	247.2853	246.1	244.5762	Q2,5	22	14.5754	15.4	14.5219
V4	225.7	228.9234	227.6	225.8319	Q2,6	15	11.1498	12.4	10.9815
V5	225.2	228.3218	226.7	225.3523	Q2,3	-11.9	-12.4951	-12.3	12.6194
V6	228.9	232.7474	231	229.8687	Q3,2	10.2	5.8985	5.7	6.1907
P1,2	31.5	30.408	28.7	30.4649	Q3,5	23.9	22.4942	23.2	22.6369
P1,4	38.9	44.8887	43.6	44.9101	Q3,6	58.3	58.5856	60.7	58.6578
P1,5	35.7	36.8059	35.6	36.7778	Q4,1	-14.3	-21.1831	-19.9	21.1814
P2,1	-34.9	-29.456	-27.8	-29.488	Q4,2	-44.3	-44.6651	-45.1	44.7199
P2,4	32.8	32.5003	33.1	32.5835	Q4,5	-17.4	-5.3576	-4.9	-5.3552
P2,5	17.4	15.5666	15.5	15.5202	Q5,4	-1.5	-2.4681	-2.8	-2.2613
P2,6	22.3	26.0293	26.2	25.9499	Q5,1	-17.5	-13.7653	-13.5	13.5474
P2,3	8.6	3.0576	2.9	3.0344	Q5,2	-22.2	-17.3573	-18	-17.18
P3,2	-2.1	-3.0157	-2.9	-2.9898	Q5,3	-29.9	-25.5853	-26.1	25.5395
P3,5	17.7	19.014	19.1	19.0379	Q5,6	-0.8	-9.7821	-9.7	-9.7472
P3,6	43.3	43.351	43.8	43.3699	Q6,5	2.9	3.9017	3.9	4.0244
P4,1	-40.1	-43.7371	-42.5	-43.7262	Q6,2	-22.3	-14.9249	-16	-14.6096
P4,2	-29.8	-31.0509	-31.6	-31.0893	Q6,3	-51.1	-56.0523	-57.9	-55.9627
P4,5	0.7	4.1884	4.1	4.12	P1	113.1	112.1026	107.9	112.1527
P5,4	-2.1	-4.149	-4	-4.0801	P2	48.4	47.6977	50	47.6
P5,1	-36.6	-35.6778	-34.5	-35.6253	P3	55.1	59.3493	60	59.418
P5,2	-11.7	-15.0952	-15	-15.0419	P4	-71.8	-70.5997	-70	-70.6955
P5,3	-25.1	-17.9697	-18	-17.9652	P5	-72	-71.4544	-70	-71.2872
P5,6	-2.1	1.4373	1.6	1.4253	P6	-72.3	-69.2772	-70	-69.1741
P6,5	1	-1.3879	-1.6	-1.3741	Q1	20.2	19.0641	16	19.4187
P6,2	-19.6	-25.481	-25.7	-25.3951	Q2	71.9	70.3058	74	70.2177
P6,3	-46.8	-42.4083	-42.8	-42.4049	Q3	90.6	86.9783	89.6	87.4853
Q1,2	-13.2	-14.1787	-15.4	-14.0847	Q4	-71.9	-71.2058	-70	-71.2566
Q1,4	21.2	21.5661	20.1	21.7989	Q5	-67.7	-68.9581	-70	-68.2754
Q1,5	9.4	11.6767	11.3	11.7045	Q6	-60.9	-67.0755	-70	-66.5479

*: kV for Voltmeters, MW for Wattmeters and MVAr for Varmeters.

TABLE 3. RESULTS OF ES FOR 14 BUSES IEEE SYSTEM

Measurement	**Measured data**	**Final Estimated data**	**Actual data**	**Optimal Estimated data**	**Measurement**	**Measured data**	**Final Estimated data**	**Actual data**	**Optimal Estimated data**
	Pu	pu	pu	Pu		pu	pu	pu	pu
V_1	1.05	1.0613	1.06	1.0603	$Q_{10,11}$	-0.015	-0.0147	- 0.0162	-0.0173
V_3	1	1.0136	1.01	1.0128	$Q_{13,14}$	0.016	-0.0004	0.0175	-0.0001
V_5	1.04	1.0237	1.02	1.0228	$Q_{5,1}$	0.022	0.0118	0.0223	0.0128
V_8	1.09	1.09	1.09	1.09	$Q_{4,2}$	0.0288	0.0185	0.0302	0.0191
V_9	1.05	1.0569	1.056	1.0558	$Q_{5,4}$	-0.1332	-0.1251	-0.142	-0.125
V_{11}	1.057	1.0588	1.057	1.0582	$Q_{7,4}$	0.1032	0.0956	0.1138	0.0963
V_{13}	1.03	1.0414	1.05	1.0401	$Q_{9,4}$	0.0159	0.0112	0.0173	0.0109
$P_{1,2}$	1.4276	1.4441	1.5688	1.4453	$Q_{11,6}$	-0.0313	-0.025	- 0.0344	-0.0205
$P_{2,5}$	0.4072	0.3809	0.4152	0.3816	$Q_{12,6}$	-0.0221	0.0117	- 0.0235	-0.004

Measurement	Measured data	Final Estimated data	Actual data	Optimal Estimated data	Measurement	Measured data	Final Estimated data	Actual data	Optimal Estimated data
$P_{5,6}$	0.4076	0.4153	0.4409	0.4145	$Q_{14,9}$	-0.0316	-0.0491	-0.0336	-0.0486
$P_{6,13}$	0.1714	0.1665	0.1775	0.1721	P_2	0.1674	0.1792	0.183	0.1785
$P_{7,8}$	0	0	0	0	P_4	-0.4763	-0.4737	-0.478	-0.4739
$P_{12,13}$	0.0158	0.0303	0.0161	0.027	P_6	-0.1021	-0.0991	-0.112	-0.0986
$P_{13,14}$	0.0518	0.0517	0.0564	0.0534	P_9	-0.2939	-0.2933	-0.295	-0.2936
$P_{3,2}$	-0.6648	-0.6632	-0.7091	-0.6624	P_{10}	-0.0833	-0.084	-0.09	-0.0839
$P_{4,2}$	-0.5043	-0.5081	-0.5445	-0.5086	P_{12}	-0.0601	-0.0554	-0.061	-0.0568
$P_{7,4}$	-0.2581	-0.2544	-0.2807	-0.2552	Q_2	0.3042	0.2938	0.3086	0.294
$P_{9,4}$	-0.1597	-0.1505	-0.1608	-0.1511	Q_3	0.0571	0.0393	0.0608	0.0401
$P_{11,6}$	-0.0661	-0.0629	-0.073	-0.0589	Q_6	0.051	0.0554	0.0523	0.0553
$P_{10,9}$	-0.0495	-0.0463	-0.0521	-0.0431	Q_9	-0.1606	-0.1648	0.1762	-0.1643
$Q_{2,5}$	0.0108	0.0131	0.0117	0.0127	Q_{14}	-0.048	-0.0478	-0.05	-0.0476

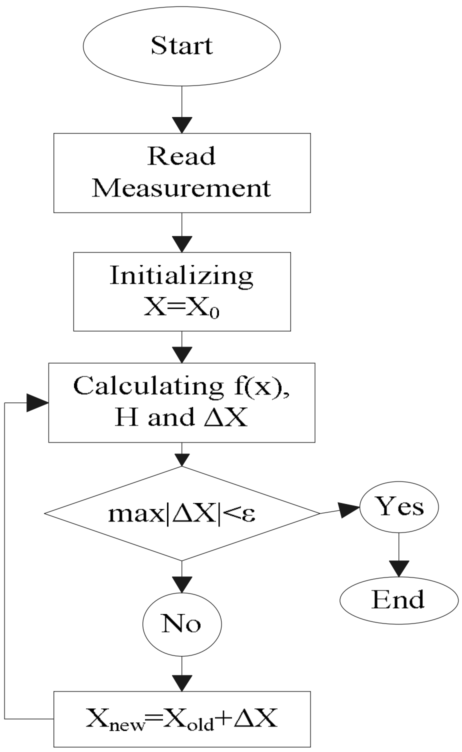

Figure 1. State estimation Algorithm by WLS method

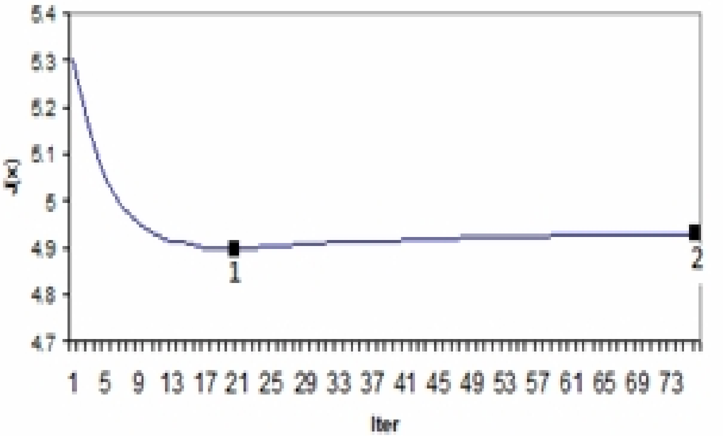

Figure 2. Values of *J(x)* for different stages of WLS iterative algorithm for a hypothetical network

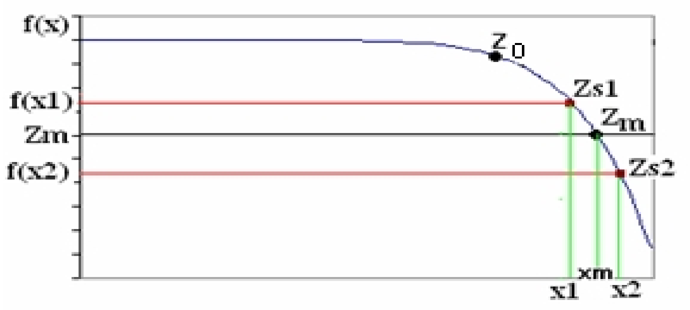

Figure 3. Analyze examples of the possibility of the *J(x)* value increased by reducing max*|Δx|*

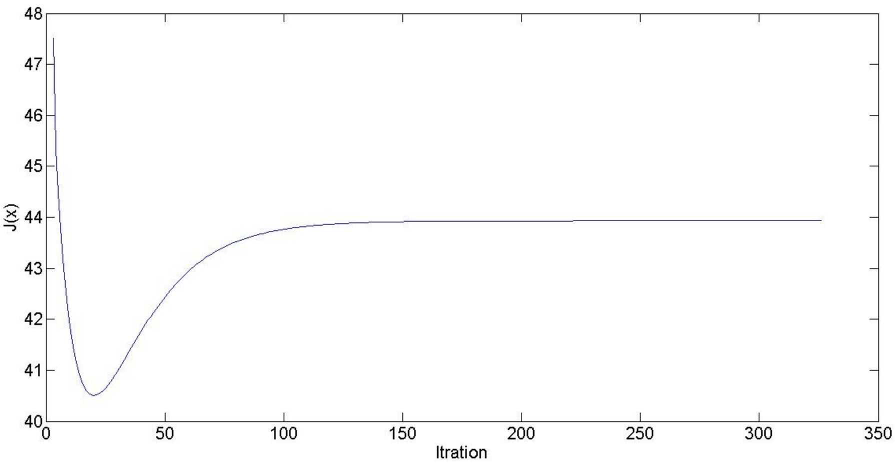

Figure 4. *J(x)* Values at different stages of state estimation optimization algorithm for 7-buses System Reference [15]

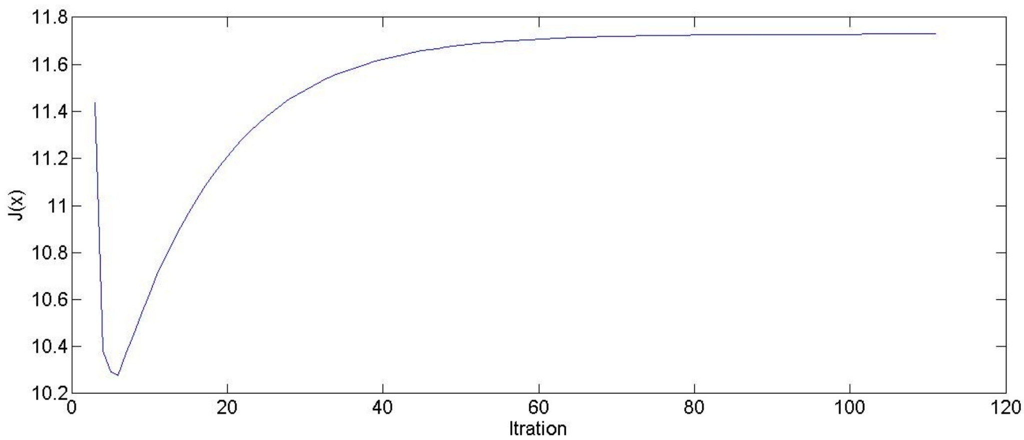

Figure 5. *J(x)* Values at different stages of state estimation optimization algorithm for IEEE 14 bus test system

CONCLUSION

Power system state estimation are performed in different intelligent techniques and mathematical methods, but experience revealed that the mathematical methods have high accuracy than the intelligent methods and among the mathematical methods the least square error method, WLS, is more accurate than the other methods. In AC systems, WLS is usually optimized in iterative method and the stopping condition of algorithm is that the maximum absolute state variables changes would be less than a tiny number. It is expected that the least square error would be obtained in the last step, but the analysis made in this paper showed that the least square error may be created in the step other than final step, and therefore the best system estimate state variables can be in the non-final stage. On this basis, it was proposed that square error value in each step and its corresponding state variables to be saved and after ending the iterative process, the least square error and its corresponding state selected as the state estimated amounts which this could increase the accuracy of the estimators.

REFERENCES

1. Xiang Nian-de, Wang Shi-ying and Yu Er-keng, (1982).“A New Apprroach for Detection and Identification of MULTIPLE Bad Data in Power System State Estimation”, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-101, No. 2, February , pp 454-462.
2. S. Naka, T. Genji, T. Yura and Y.Fukuyama, (2002). “A hybrid particle swarm optimization for distribution state estimation,” IEEE Transactions Power System, vol. Pas 18, Nov , pp 55-57.
3. D.M.V. Kumar, S. C. Srivastava, S. Shah and S. Mathur, (1996) “Topology Processing and Static State Estimation Using Artificial Neural Networks” Proc. Inst. Elect. Eng. C, vol 148, Jan, , pp 51-60.
4. M. Shahidehpour and D Labudda,(1993). “ A Fazy Molti-objective Approach to Power System State Estimation”, Fourth International Symposium on Expert System Application to Power System, pp218-223.
5. D Chauhan and D Singh and J P Pandey,(2005). “Topology Identification, Bad Data Processing, and State Estimation Using Fuzzy Pattern Matching”, IEEE Transaction Power System, vol. Pas 20, pp 1570-1579.
6. Yeojun Yoon, (2005). “Study of the Utilization and Benefits of Phasor Measurement Units for Large Scale Power System State Estimation,” MS Thesis, Texas A&M University.
7. A.Abur and A.Gomez Exposito,(2004). “Power System State estimation: Theory and Implementation”, New York, Marcel Dekker.
8. D. Falcao and S.M. Asis, (1988). “Linear Programming State Estimation Error Analysis and Gross Error Identification” IEEE Transaction Power System, vol. Pas 3, pp 809- 815.
9. A. Monticelli, (2000). “Electric power system state estimation”, IEEE Transaction Power System, vol. Pas 88, Feb. pp. 262–282.
10. A. Abur and M.Celik,(1991). “A Fast Algorithm for the Weighted Least Absolute Value State Estimation”, IEEE TransACTION Power System, vol. Pas 6, Feb 1991, pp 1-8.
11. F. C. Schweppe, J. Wildes, D.B.Rom, (1970). “Power System Static-State Estimation”, Part 1,2 and 3, IEEE Transaction Power Application and Systems, Vol. Pas 89, Jan 1970, pp.120-135.
12. L. Mili, Th. Van Cutsem, and M. Ribbens-Pavella,(1985). “ Bad Data Identification methods in Power System State Estimation: a comparative study,” IEEE Power Transactions on Power Apparatus and Systems, vol. PAS- 104, no. 11, pp. 3037-3049, November 1985.
13. E. Handschin, F.C. Schweppe, J. Kohlas, and A. Fiechter (1975).“ Bad data analysis for power system state estimation,” IEEE Transactions on Power Apparatus and Systems, vol. PAS-94, no. 2, pp. 329-337, March/April 1975.
14. A. Gracia, A. Monticelli, and P. Abreu, (1979).“ Fast decoupled state estimation and bad data processing,” IEEE Power Transactions on Power Apparatus and Systems, vol. PAS-98, no. 5, pp. 1645-1652.
15. Allen J. Wood and Bruce F. Wollenberg,(1996). “Power GenerationT Operation and Control”, Wiley-Interscience, 2 editions.
16. Slobodan Paji, (2007).Power System State Estimation and Contingency Constrained Optimal Power Flow - A Numerically Robust Implementation Worcester Polytechnic Institute PHD Dissertation.

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