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## **Optimum Tilt Angle for Fixed-Array Solar Panels at a Constant Latitude of 29° to Receive the Maximum Sunlight**

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### **ABSTRACT**

A key parameter affecting the performance of solar panels is tilt angle. Finding the optimum tilt angle helps receive the maximum energy monthly, quarterly and yearly. In fact, changes in tilt angle affect the amount of solar radiation reaching the surface of panels. Employing a mathematical model, this study estimates the solar radiation incident on the surface of a tilted panel. Accordingly, optimum tilt and azimuth angles of solar panels are defined for latitude 29 at different locations around the world. Target locations are selected based on the global position as well as the potential to develop a solar power plant. The results revealed that the optimum tilt angle is somewhat different for different months. In addition, we found that by selecting the optimum tilt angle for solar panels at a month, the total solar radiation would be approximately equal to the maximum value found based on daily changes in tilt angle. In this case, a tilt array with optimum angle improves the incident annual solar radiation on panels up to 30% compared to a horizontal solar panel.

Keywords: solar radiation, solar array, optimum tilt angle, azimuth angle

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### **INTRODUCTION**

Solar panel is the basic component of solar energy conversion in a photovoltaic system. The power generated by solar photovoltaic system is associated with the solar energy radiated on modules. Thus, to estimate the electrical power generated by solar modules, we must consider the architecture of solar system on a tilted surface at various azimuths so to receive the maximum solar radiation. There are many articles providing information on solar energy gathered by modules. They argued that the amount of incident solar energy depends on various external factors such as solar radiation, temperature, load consumption, and panels' orientation [1]. To maximize solar power, the only flexible factor is tilt angle of panels. Particularly, for a fixed module, the collected solar energy is highly dependent on tilt and azimuth angles of solar panels. Since only about 20% of the incident solar radiation is scattered light, fixed solar panels do not have a significant negative effect on efficiency. This is while the primary benefit of a tracking system is to collect solar energy for the longest period of the day, and with the most accurate alignment as the Sun's position shifts with the seasons. [3]. However, operating and maintenance costs of a tracking system are high and they cannot be always used. Therefore, it is a best choice in most cases to install solar panels at an optimum tilt angle. Different articles proposed different values as optimum tilt angle for fixed solar panels. For example, Qiu and Riffat suggested the tilt angle of the solar collector set within the optimum tilt angle of  $\pm 10^\circ$  [4]. Other studied suggested  $\Sigma = \varphi \pm 20^\circ$  [5],  $\Sigma = \varphi \pm 8^\circ$  [6],  $\Sigma = \varphi \pm 5$  [7] and  $\Sigma = \varphi \pm 21$  [8]; where  $\varphi$  is latitude of the target location and symbols + and - indicates winter and summer, respectively.

The objective of this study is to define the optimum tilt angle of a fixed solar array located at Shiraz to receive the maximum solar energy. We then compare it with environmental conditions of other leading

regions in the world such as Texas in USA, Timimoun in Algeria, and Tibet in China. Table 1 lists geographical and climatic characteristics of these regions [9].

Table 1: Geographical and annual climatic characteristics of different regions [10-12].

	Shiraz (Fars)	San Antonio (Texas)	Timimoun (Algeria)	Lhasa (Tibet)
Latitude	+29.63 (29°37'48"N)	+29.42389 (29°25'26.004"N)	+29.26 (29°15'36"N)	+29.65 (29°39'00"N)
Longitude	+52.57 (52°34'12"E)	-98.49333 (98°29'35.988"W)	+0.23 (0°13'48"E)	+91.1 (91°06'00"E)
Altitude	1600 m	210 m	320 m	3900 m
Temperature, °C (average of 10 years)	14.81	19.79	22.77	-1.17
Insolation, kWh/m <sup>2</sup> /day	5.21	4.50	5.62	4.28
No. of Sunny day (average of 10 years)	259.1	123.4	287.2	175.9
No. of partly cloudy day(average of 10 years)	83	186.9	62.3	169

## CALCULATIONS

Defined mathematical models calculate incident energy on tilted panels using data on energy radiated on a horizontal surface. Total daily radiation (monthly-average) on a tilted surface is sum of direct solar radiation  $I_B$ , diffuse radiation  $I_D$ , and reflected radiation  $I_R$ . Therefore,  $I_T$  can be defined as [13, 14]:

$$I_T = I_B + I_D + I_R \quad (1)$$

Direct radiation ( $I_B$ ) at a certain location can be calculated using Eq. 2 to 5:

$$I_B = A e^{-km} \quad (2)$$

$$A = 1160 + 75 \sin \left[ \frac{360}{365} (n - 75) \right] \text{ w/m}^2 \quad (3)$$

$$\sin \left[ \frac{360}{365} (n - 75) \right] 0.035 \quad K = 0.174 + \quad (4)$$

$$m = 1 / \sin \beta \quad (5)$$

Where  $A$  is the energy from the sun in the atmosphere;  $K$  is depth of radiation in the atmosphere;  $m$  is air masses;  $n$  is day number in a year; and  $\beta$  is the angle of elevation of the sun.

The solar energy absorbed by a solar module from the direct radiation ( $I_{BC}$ ) can be calculated using Eq. 6 and 7:

$$I_{BC} = I_B \cos \theta \quad (6)$$

$$\theta = \cos^{-1} [\cos \beta \cos (\varphi_s - \varphi_c) \sin \Sigma + \sin \beta \cos \Sigma] \quad (7)$$

Where  $\theta$  is the angle between the vector perpendicular to the panel and the direct radiation;  $\varphi_s$  is the azimuth of the sun towards the south;  $\varphi_c$  is azimuth of the panels towards the south; and  $\Sigma$  is the tilt angle of the panel. Defuse solar radiation ( $I_{DH}$ ) that reaches the ground level is calculated using Eq. 8 and 9; and the energy absorbed by a tilted module from defuse solar radiation is calculated through Eq.10 [15]:

$$I_{DH} = C I_B \quad (8)$$

$$C = 0.095 + 0.04 \sin \left[ \frac{360}{365} (n - 100) \right] \quad (9)$$

$$I_{DC} = I_{DH} \left( \frac{1 + \cos \Sigma}{2} \right) = C I_B \left( \frac{1 + \cos \Sigma}{2} \right) \quad (10)$$

Where  $C$  (in Eq. 8 and 9) denotes atmospheric scattering coefficient [15]. Now we need to calculate the reflected component of the solar radiation. The energy absorbed by a solar module from the reflected radiation ( $I_{RC}$ ) is obtained through Eq.11.

$$I_{RC} = \rho (I_{BH} + I_{DH}) \left( \frac{1 - \cos \Sigma}{2} \right) = \rho I_B (\sin \beta + C) \left( \frac{1 - \cos \Sigma}{2} \right) \quad (11)$$

Where  $\rho$  is the reflection coefficient<sup>1</sup>. Residual energy reflected from the ground depends on the tilt angle of the panel,  $\Sigma$ . Thus, the total energy absorbed by the module from the sun can be calculated from Eq.12 to 15.

$$I_T = I_{BC} + I_{DC} + I_{RC} = I_B \cos \theta + C I_B \left( \frac{1 + \cos \Sigma}{2} \right) + \rho I_B (\sin \beta + C) \left( \frac{1 - \cos \Sigma}{2} \right) \quad (12)$$

$$\beta = \sin^{-1} [\cos \varphi \cos \delta \cos H + \sin \varphi \sin \delta] \quad (13)$$

<sup>1</sup> Earth reflection coefficient ranges from 0.8 for fresh snow to 0.1 for asphalt. It is usually defined for typical ground and grassland equal to 0.2.

$$\delta = 23.45 \sin\left[\frac{360}{365}(284 + n)\right] \tag{14}$$

$$H = 15 \times (12 - ST) \tag{15}$$

Where,  $\delta$  is the declination angle of the sun;  $\beta$  is the elevation angle of the sun;  $\varphi$  is the latitude of the location; H is the hour angle<sup>2</sup>; and ST is daytime [13, 15]. Using equations 1 to 15 and the computational program of *Mathematica*<sup>3</sup>, we calculated incident direct, diffuse and reflected solar radiations and finally total solar radiation incident on a tilted panel for tilt angles ranging from 0 to 90 degrees (with step increments of 5 degrees) at each day and month of year. Average solar reflection coefficient ( $\rho$ ) and air quality index were defined 0.22 [12] and 0.63 [12, 16], respectively.

**RESULTS**

According to the relationships presented in the previous section, the two factors affecting solar radiation absorption are panel azimuth angle ( $\theta_c$ ) and panel tilt angle ( $\Sigma$ ). By optimizing these two factors, we can store more annual energy. As a general rule, designers consider panel tilt angle near to the latitude of the region and panel azimuth angle equal to 180 degrees (north-south direction). First, a fixed-tilted array with  $\theta_c = 180$  is considered, then  $\Sigma$  is changed from horizontal (zero degrees) to 90 degrees to find the optimum angle ( $\Sigma_{opt}$ ) for each month. These values are listed in table 2. Finally, for comparison purposes, the last column of Table 2 lists values obtained in other studies.

Table 2: Optimum tilt angle for solar panels for each month in Shiraz

Month	$\Sigma_{opt}$ (degrees)	Total solar Radiation (KWh/m <sup>2</sup> .day)	Other $\Sigma_{opt}$ [17]
Jan	55	8.168	54.64
Feb	45	8.344	40.48
Mar	30	8.391	26.22
Apr	14	8.590	13.34
May	0	8.956	1.31
Jun	0	9.057	-5.23
Jul	0	8.929	-2.07
Aug	10	8.525	8.79
Sep	30	8.307	24.96
Oct	45	8.264	39.57
Nov	55	8.121	51.01
Dec	60	8.001	57.50

As obvious, the mean optimum tilt angle for cold months is more than warm months. The optimum tilt angle of a solar panel is 55 degrees in January with the total incident radiation of 8.168 KWh/m<sup>2</sup>.day while this value is zero in June and July with the incident radiations equal to 9.057 and 8.929 KWh/m<sup>2</sup>.day, respectively. Another notable point is the greater total solar radiation in June and July compared to other months of the year, which was predictable. This is while the minimum total solar radiation happens in December (8.001 KWh/m<sup>2</sup>.day) and January (8.168 KWh/m<sup>2</sup>.day). All results obtained in this study are in close agreement with results obtained by Talebzadeh et al. for Shiraz [17]. Table 2 shows the mean optimum tilt angle for each season and the year.

Table 2: Mean optimum tilt angle of a panel for seasons and the years in Shiraz

	Mean Optimum Tilt Angle	Mean Absolute Deviation	Corresponding Values at Other Papers [17]
Winter	43.33	14.58	40.45
Spring	5	23.75	3.14
Summer	13.3	15.45	10.56
Fall	53.3	24.55	49.36
Annually	28.75	----	25.88

A remarkable point from Table 2 is the notable approximation of mean annual optimum angle (28.75 degrees) to latitude of the area (28.75 degrees). Based on data reported in some studies and as a general

<sup>2</sup> The hour angle is the number of degrees that the earth must rotate before the sun will be directly over your local meridian (line of longitude).

<sup>3</sup> Wolfram Mathematica 10

rule, the optimum tilt angle for a fixed array panels is close to the latitude of the area. The results obtained in this study confirm this rule [18].

Figures 1-a and 1-b show mean daily radiation on a tilted panel at different tilt angles for the first half of the year and for the second half of the year, respectively. Clearly, the optimum tilt angle differs from month to month. For example, the optimum tilt angle in Shiraz is 45 degrees in October and reaches a maximum of 60 degrees in December. There is a unique optimum angle ( $\Sigma_{opt}$ ) for each month for which the solar radiation is the maximum possible value during a year. The optimum tilt angle decreases for next winter months reaching 45 degrees in February and ultimately 30 degrees in March.

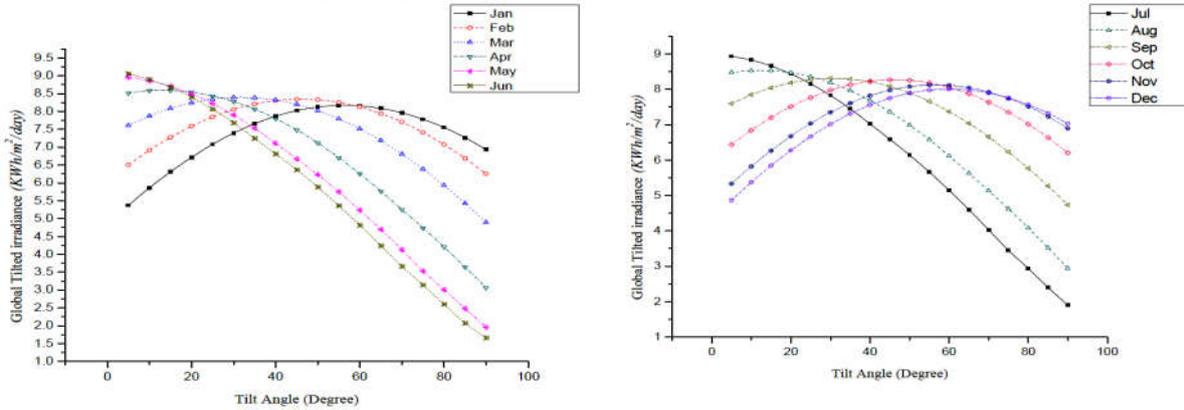


Figure 1: Mean daily solar radiation on the surface of a tilted panels with different angles (a) for the first 6 months of the year, and (b) for the second 6 months of the year.

Figure 2 shows changes in the optimum tilt angle of fixed array panels in different months of the year. As already mentioned, the maximum tilt angle belongs to the cold months, while it is near zero for summer. It means that to receive the maximum power from solar modules during the summer, we can install solar panels horizontally.

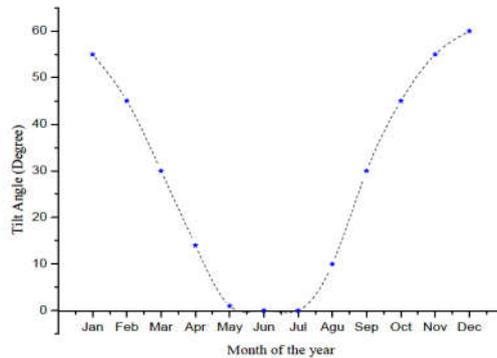


Figure 2: Trend of changes in tilt angle of solar panels at different months, Shiraz.

In addition, changes in mean solar radiation per month in Shiraz is plotted in Figure 3. As mentioned, the mean solar radiation reaches its highest value at June while the corresponding panel tilt angle is the lowest. This is clearly shown in Figures 2 and 3.

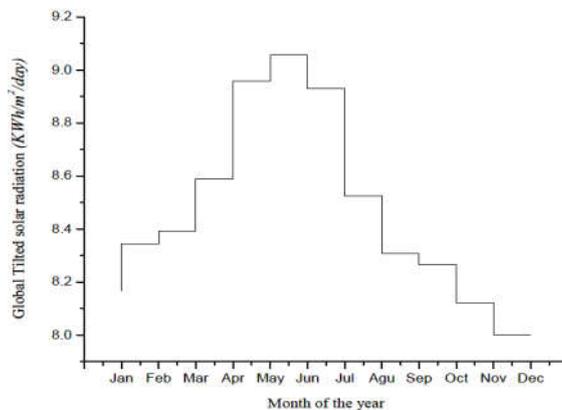


Figure 4: Mean daily solar radiation.

## DISCUSSION AND CONCLUSION

The mean optimum tilt angle for solar panels located at Shiraz, Iran is 43.13 degrees during the winter and 5 degrees during the summer. Annual optimum tilt angle depends on several factors, including latitude and climatic conditions of the region. This value was calculated equal to 28.75 degrees which was very close to the latitude of the region. This fixed angle is considered as the annual optimum angle.

By optimizing the tilt angle of solar panels, the mean total solar radiation incident on a tilted panel would be almost constant throughout the year and thus more solar energy would be absorbed.

Compared to other regions, Shiraz has a higher potential to take advantage of fixed-array solar panels. The reason is that both factors affecting the intensity of received radiation, i.e. direct solar radiation and mean annual temperature, are simultaneously in a better condition for this area compared to other areas such as San Antonio in America, Timimoun in Algeria, and Lhasa in Tibet. Although the maximum radiation is measured in Timimoun of African, high mean annual temperature of the area has a negative effect on efficiency of solar panels. In terms of temperature, Tibet is a suitable location to install panels; however, the intensity of solar radiation is minimum which leads to the poor efficiency of solar panels in absorbing the maximum solar power.

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