



## **Numerical analysis of dissipation of pore water pressure using Natural Element Method**

**Farzane Farahbakhsh<sup>1</sup>, Mohammad Mohsen Toufigh<sup>2</sup>, Fazlollah Soltani<sup>3</sup>, Vahid Toufigh<sup>4</sup>**

<sup>1</sup>M.S.Student, Department of civil engineering, Kerman graduate university of advanced technology, Kerman, Iran

<sup>2</sup> Professor, Department of civil engineering Bahonar university, Kerman, Iran

Assistant Professor, Department of civil engineering

Kerman graduate university of advanced technology, Kerman, Iran

Assistant Professor, Department of civil engineering

Kerman graduate university of advanced technology, Kerman, Iran

### **ABSTRACT**

*The Natural Element Method (NEM) is a meshfree numerical method for solving partial differential equations. In this paper NEM is used to solve a consolidation problem based on Biot consolidation theory. This problem is also solved using Plaxis in order to compare results with NEM. Since natural element method have smoother shape functions comparing to Finite Element Method, more accurate results are expected. The excess pore water pressure in this method is dissipated faster than FEM that can affect on scheduling and cost estimation in big projects.*

**Keywords:** meshless, Natural Element Method, Biot consolidation theory, water pore pressure

### **INTRODUCTION**

Consolidation is encountered in various areas of soil mechanics. The phenomenon needs to be analyzed in design of footings, pile foundation and embankments. Consolidation theory was performed by Terzaghi [1]. This theory is based on some simplifier that makes some errors. The major error is in the assumption of being one dimensional consolidation and water flow. Biot (1941) solved this problem by assuming three dimensional consolidation and flow [3]. Various numerical solutions are used to solve this. One of the most important of these solutions is Finite Element Method. FEM is a powerful method in solving different kinds of problems, however it has some limitation, such as discretization and discontinuity in stress calculation. McNamee, J. Gibson, R.E had solved consolidation problems with analytical approach [4]. Ai, Z.Y. Wang, Q.S. proposed a new analytical solution for axisymmetric Biot's consolidation [6].

Meshfree introduces the domain with nodes. These nodes do not need any special connection with each other. Moreover, the meshfree method has a lot of branches such as EFG, PIM, NEM, ... . NEM is chosen because of its advantage in boundary condition satisfaction.

### **natural element method**

Over the last decade a number of meshless methods have been proposed. they can sub divided in accordance with the definition of the shape function and/or in accordance with the minimization method of approximation. The minimization may be via strong form (as in the point collocation approach) or a weak form (as in the Galerkin method or NEM) [8]. In this paper, Natural Element Method is used to solve a consolidation problem. The NEM interpolant is constructed on the basis of the underlying Voronoi tessellation. The Delaunay triangulation is the topological dual of the Voronoi diagram. Within the context of natural neighbor circumcircle. In Fig.1, some of the important geometric constructs associated with a set of nodes are illustrated [7]. Consider a set of distinct nodes  $N = \{n_1, n_2, n_3, \dots, n_M\}$  in  $R^2$ . The first order Voronoi diagram of the set  $N$  is a subdivision of the plane into regions  $T_i$  (Voronoi cells) given by

---

$$(1)$$

$$T_i = \{X \in \mathbb{R}^2 : d(X, X_i) < d(X, X_j) \forall j \neq i\}$$

Where  $d(X_i, X_j)$ , the Euclidean metric, is the distance between  $X_i$  and  $X_j$ .

The voronoi diagram for a set of seven nodes is shown in Fig.2a, and a point  $x$  is introduced into the voronoi diagram of the set  $N$ . if  $x$  is tessellated along with the set of nodes  $N$ , then the natural neighbor coordinates of  $x$  are those nodes which form an edge of triangle with  $x$  in the new triangulation. The natural neighbor coordinates of  $x$  with respect to a natural neighbor  $I$  is defined as the ratio of the area of overlap of their Voronoi cells to the total area of overlap of their Voronoi cells to the total area of the Voronoi cell of  $x$  ( see Fig. 2b) [7]:

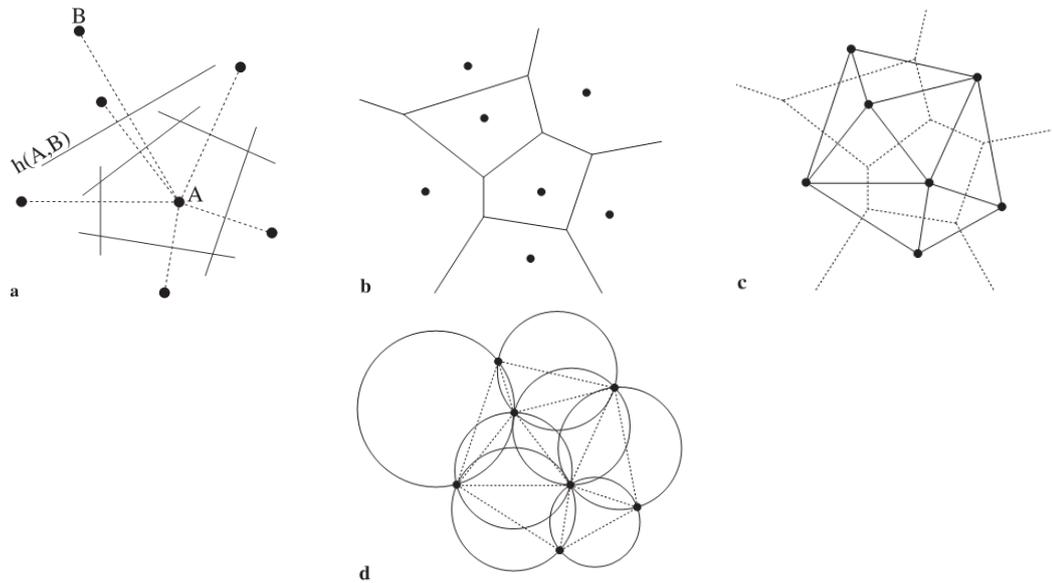


Fig. 1a-d Geometric structures for a set  $N$  of seven nodes. **a** Voronoi cell for node  $A$ , **b** Voronoi diagram  $V \dots N \dagger$ , **c** Delaunay triangulation  $DT \dots N \dagger$ , **d** natural neighbor circumcircle

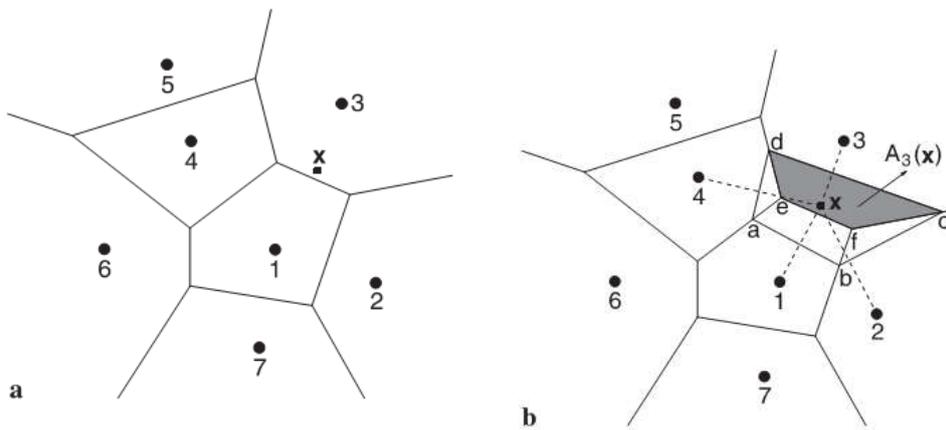


Fig. 2a,b .Construction of natural neighbor coordinates. **A** Original Voronoi diagram and  $x$ , **b** first-order and second-order Voronoi cells about  $x$

$$\Phi_I(X) = A_I(X) / A_J(X) \tag{2}$$

Where  $I$  ranges from 1 to  $n$ . If the point  $x$  coincides with a node ( $x=x_i$ ),  $\Phi_I(X) = 1$ , and all other shape functions are zero. The properties of positivity, interpolation, and partition of unity directly follow:

$$\sum_{I=1}^n \Phi_I(X) = 1 \quad X \in \Omega, \quad 0 \leq \Phi_I(X) \leq 1 \quad \Phi_I(X_j) = \delta_{IJ} \tag{3}$$

### Evaluation of Natural Element Method

The main advantage of this method over the previously used meshless methods is the use of Voronoi diagrams to define the shape functions, which yields a very stable partition. The added advantage is the capability for nodal data interpolation, which facilitates a mean to impose the essential boundary conditions[8].

Finally, all the shape functions, including the FEM shape function, may be defined as partition of Unity approximation. Several other shape functions may also be developed using this concept. See Fig. 3, for a quick comparison of shape functions.

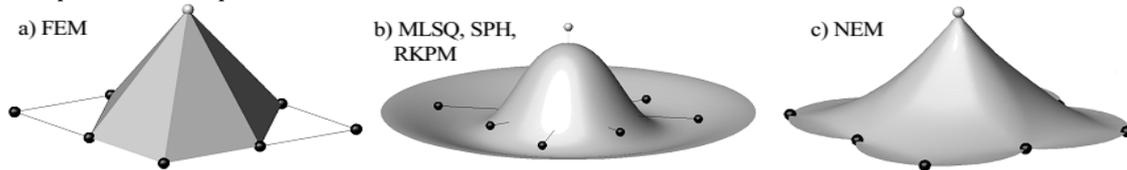


Fig3. Shape function for a regular node distribution

Some of the critical weakness of pervious mesh less methods are[8]:

1. In some cases it is difficult to introduce the essential boundary conditions.
2. For some methods it is laborious to evaluate the shape function derivatives.
3. Often, too many Gauss points are needed to evaluate the weak form.
4. The shape functions usually have a continuity order higher than  $C^0$ . This decreases the convergence of the approximation and makes it more difficult to introduce discontinuities such as those due to heterogeneous material distributions.
5. Some of the methods do not work for irregular point distributions, or need complicated node connectivity to give accurate results.

### Consolidation theory

According to the well-known Biot's consolidation theory (Biot 1941), the primary variables of a fully saturated porous media are the solid skeleton displacement  $\mathbf{u}$  and the water pore pressure  $\mathbf{p}$ . the differential equations of solid part and fluid part are stated as

$$\mathbf{L}^T \boldsymbol{\sigma} - \alpha \mathbf{p}_{,i} + \mathbf{b} = \mathbf{0} \quad \text{in } \Omega \quad (4)$$

$$\alpha \dot{\boldsymbol{\varepsilon}} + \frac{1}{K_c} \dot{p} + \mathbf{q}_{i,i} = \mathbf{0}$$

Where  $\boldsymbol{\sigma}$  is the total stress in the solid and fluid mixture,  $\mathbf{b}$  is the body force,  $\alpha$  is Biot willis coefficient,  $\dot{\boldsymbol{\varepsilon}}$  denotes the volumetric strain rate of the skeleton without volumetric change of grains,  $K_c$  is the combined compressibility parameter,  $\mathbf{q}_i$  is the fluid flow rate defined as

$$\mathbf{q}_i = -k(p_{,i} - \rho_f g) \quad (5)$$

Where  $k$  is permeability.

Eqs. (4) constitute the set of coupled equations governing the consolidation process of porous medium in a domain  $\Omega$  with boundary  $\Gamma$ . The corresponding weak forms for these two equations are

$$\int_{\Omega} \delta \boldsymbol{\varepsilon} \mathbf{D} \boldsymbol{\varepsilon} \, d\Omega - \int_{\Omega} \delta u_{i,i} \alpha p \, d\Omega = \int_{\Omega} \delta u_i \rho b \, d\Omega + \int_{\Gamma} \delta u_i \bar{T}_i \, d\Gamma \quad (6)$$

$$\int_{\Omega} \delta p_{,i} k p_{,i} \, d\Omega + \int_{\Omega} \delta p \alpha \dot{\boldsymbol{\varepsilon}}_{i,i} \, d\Omega + \int_{\Omega} \frac{\delta p \dot{p}}{K_c} \, d\Omega = \int_{\Omega} \delta p_{,i} k \rho_f g \, d\Omega + \dots \quad (7)$$

### Verification

Figure 4 is showing a clay layer with specific properties. Linear elastic behavior is assumed. Shape functions are determined using Watson Algorithm. Delaunay triangles are used as background cells for numerical integration. This problem is also solved using Plaxis in order to compare results with NEM.

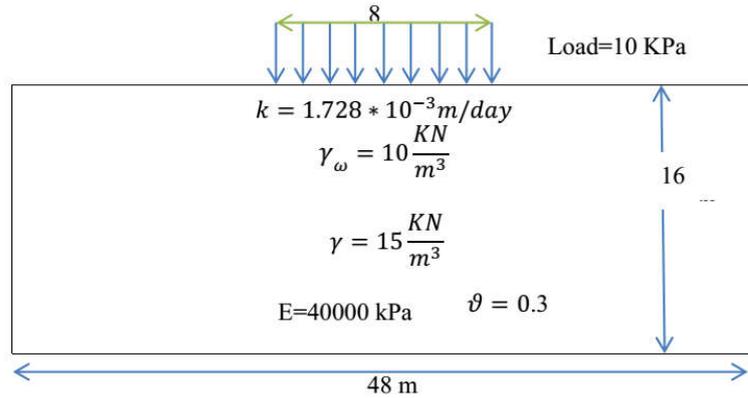


Figure 4. soil properties and layer boundary conditions

In order to compute shape function, stress and pore pressure, a code was written in Matlab. The code needs geometric parameters and mechanical properties as input. 833 nodes are used to define the domain of the problem (Fig.5). In Plaxis, 15 nodes elements are used (Fig.6)

Figure 7 is showing excess pore water pressure at different evaluations of the vertical symmetric axis.

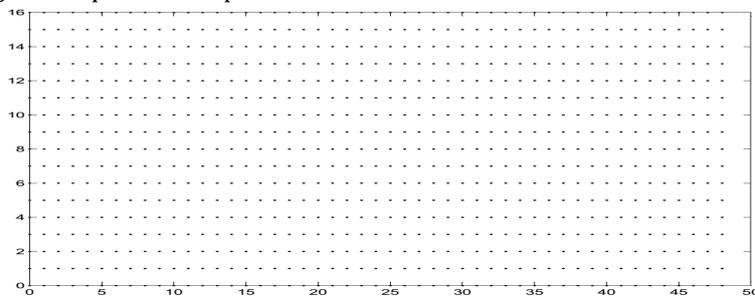


Figure 5. node generation

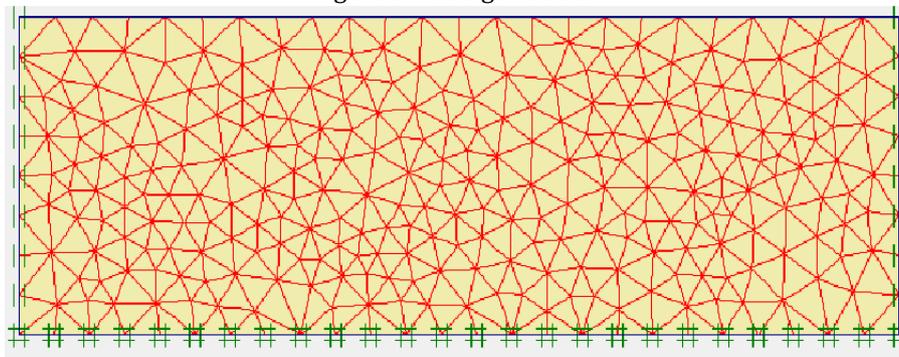


Figure 6. meshed layer in Plaxis

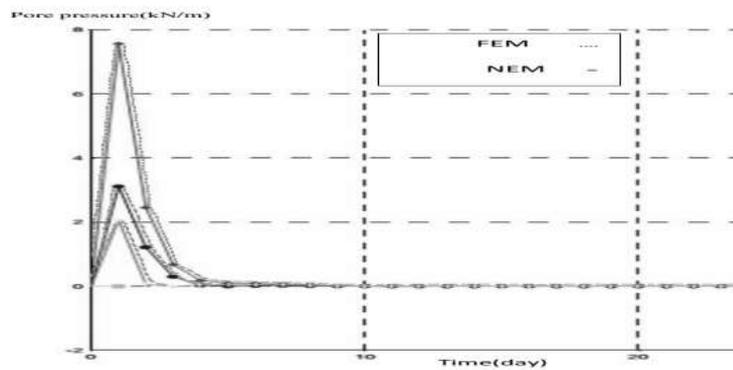


Figure 7. dissipation of excess water pore pressure

## CONCLUSION

Since natural element method have smoother shape functions comparing to Finite Element Method, more accurate results are expected.

In conclusion, the excess pore water pressure in this method is dissipated faster than FEM. If this is the case, it can extremely affect scheduling and cost estimation in big project.

## REFERENCES

1. Terzaghi, K". Die Berechnung der durchlassig -keitsziffer de tones aus demverlang derhydro -dynamischen spannung ser cheinungen". Akademie der Wissenschaften in wein, 132,pp.125-138,(1923).
2. Rendulic,L. " Porenziffer and poren wasserdruck in Tonen". Der Bainingenieur, Vol 71, No. 5153, pp. 559-564,(1936).
3. Biot, M.A "General theory of three -dimensional consolidation". Journal of Applie Physics;Vol12 : pp.156-164,(1941).
4. McNamee,J. Gibson,R.E "Plane strain and axially symmetric problems of the consolidation of a semi-infinite clay stratum." Q J Mech Appl Math13(2): pp.210-227,(1970)
5. Schiffman RL, Fungaroli AA. " Consolidation due to tangential loads". Proceedings of the 13thInternational Conference on Soil Mechanics and Foundation Engineering, vol. 1, Montreal, Canada,188-192,(1965).
6. Ai, Z.Y. Wang, Q.S. "A new analytical solution to axisymmetric Biot's consolidation of a finite soil layer."Appl. Math. Mech. -Engl. 29(12): pp.1617-1624,(2008).
7. Sukumar N., Moran B., Belytshko T., " The Natural Element Method in Solid Mechanics" International Journal for Numerical Method in Engineering, 839-860. (1998).
8. Sergio, R.I and Onate, E. (2002). "Meshless finite element idea" . fifth world congress on computational mechanics, Australia.
9. Wang D, Xie P. and Lu H. ,"Meshfree consolidation analysis of saturated porous media with stabilized conforming nodal integration formulation". Interaction and Multiscale Mechanics, Vol. 6, No. 2 (2013)